Heat and mass transfer in the entrance region of the falling film: Absorption, desorption, condensation and evaporation

V.E. Nakoryakov, N.I. Grigoryeva, M.V. Bartashevich*
Kutateladze Institute of Thermophysics, Siberian Branch, Russian Academy of Sciences, 1, Acad. Lavrentiev Avenue, Novosibirsk 630090, Russia

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1. Introduction

The heat and mass transfer processes take place in various devices and proceed with a pressure-driven liquid film flow through holes or slits and liquid flowing in the form of rivulets, streams, or films with free boundaries of different shapes (flat, conical, etc.). In the spray-type devices such flows are essential. In other types of devices, including film-like devices, such flows are possible at the exit of the distributors, as well as free liquid overflow from one heat transfer surface to another (for example, free film flowing between two adjacent tubes). The liquid flow with free boundaries in the real devices are often accompanied by the various heat and mass transfer processes such as condensation, evaporation, absorption, and desorption.

The absorption heat pump is a unique device consisted of the desorber, condenser, evaporator and absorber. Over the last 40 years, most absorption researches devoted their attention (or work) to the falling films over the cooled surfaces. The detailed review of the existing publications was presented by Killion and Garibashev [5,6]. The R12-refrigerant is characterized by a low surface tension at large heat fluxes and has a tendency to form films. The aim of the present work is the analysis of the coupled heat and mass transfer during absorption, desorption, condensation and evaporation in the entrance region of the falling axisymmetric semi-infinite film surrounded by the pure vapour. The self-similar solutions for temperature and concentration in the film have been obtained. The corrections, which take into account a change in the film thickness due to the mass inflow in absorption and condensation processes and the mass loss due to desorption and evaporation have been derived.

2. Heat and mass transfer during non-isothermal absorption and desorption

It is assumed that the liquid film falls under the action of gravity with the uniform vertical velocity \( u_0 \) at the leading edge. The Cartesian coordinate system (Fig. 1) is chosen in such a way that \( x \) is the direction of the flow, and \( y \) is the direction from the free film surface to the axis of symmetry. At the initial condition at \( x = 0, u = u_0 \) the differential equation of motion \( \frac{u}{u_0} = g \) has the solution

\[
u = \sqrt{u_0^2 + 2gx}.
\]

In the absence of mass inflow/outflow (without condensation, absorption, desorption and evaporation) the continuity equation \( \frac{\partial}{\partial x} \int_0^{\infty} \rho dy = 0 \) leads to an ordinary differential equation for the film thickness:

\[
\frac{d\delta}{dx} + \frac{g}{u_0^2 + 2gx} \delta = 0.
\]

The solution of this equation under the condition \( x = 0, \delta = \delta_0 \) is...
In the case of falling film absorption, when the vapour or gas phase is stationary, the coupled heat and mass transfer is considered under the assumptions [2,3], which are used in absorption heat pumps and refrigerators.

The solution consists of two components; one of them is absorbed from the gas phase being in contact with the solution, whereas the other component is not consumed and is not added into the solution. Thus, the phase boundary is impermeable to the second component of the solution and displaces due to the entry of the absorbate into the solution.

The gas or vapour is one-component medium. The vapour pressure does not change during absorption. In the range of varying temperature and concentration during the whole absorption process, the density of solution as well as thermophysical properties and the transfer coefficients remain constant. The heat conductivity of the solution is much higher than that of the vapour. The absorption heat is released at the interface and is consumed only on heating the liquid phase (the solution).

At the phase boundary we observe equilibrium of the two-phase vapour-solution system; for the given constant pressure in the case of the binary solution, it is described by the temperature dependence of the absorbate concentration \( C = f(T) \) (Fig. 2). For small temperature and concentration intervals, this dependence can be approximated by the linear function \( C_i = k_1 + k_2 T_i \), in which the coefficients \( k_1 \) and \( k_2 \) are determined by the pressure. The heat generation during absorption leads to change in equilibrium temperature at the phase boundary and, consequently, in equilibrium concentration according to the relationship. Thus, the point with the coordinates \( (C_i, T_i) \) displaces along the line of equilibrium. At each instant of time, including the initial one, the location of the point is unknown and has to be determined.

In the coordinate system with the \( Y \) axis directed from the interface to the film depth, the processes of heat and mass transfer in the solution film are described by the system of heat conduction and diffusion equations:

\[
\begin{align*}
&u \frac{\partial T}{\partial x} = a \frac{\partial^2 T}{\partial y^2}, \\
&u \frac{\partial C}{\partial x} = D \frac{\partial^2 C}{\partial y^2},
\end{align*}
\]

with the corresponding conditions at the inlet \( x = 0 \): \( T = T_0, \ C = C_0, \ \delta = \delta_0 \).

**Nomenclature**

- \( a \) thermal diffusivity
- \( C \) absorbate (desorbate) concentration (mass fraction)
- \( C_p \) specific heat
- \( D \) diffusivity
- \( g \) gravity
- \( m_w \) mass flux density of absorbate (desorbate)
- \( P \) pressure
- \( r_a \) absorption heat
- \( T \) temperature
- \( u \) longitudinal velocity component
- \( x, y \) coordinates

**Dimensionless criteria**

- \( \text{Fr} \) Froude number
- \( K \) phase transition criterion
- \( K_a \) phase transition criterion for absorption
- \( \text{Le} \) Lewis number
- \( \text{Nu} \) Nusselt number
- \( \text{Pe} \) Peclet number

**Greek symbols**

- \( \delta \) film half thickness
- \( \gamma \) dimensionless concentration
- \( \lambda \) thermal conductivity
- \( \theta \) dimensionless temperature
- \( \rho \) density
- \( \zeta \) self-similar variable

**Indices**

- \( 0 \) initial value
- \( i \) surface
- \( e \) equilibrium

\[
\Delta_0 = \frac{\delta}{\delta_0} = \left( 1 + 2 \text{Fr} \frac{x}{\delta_0} \right)^{-1/2},
\]

where \( \text{Fr} = \frac{g \delta_0}{u^2} \).
and the conjugate conditions at the interphase boundary

\[ y = 0, \quad C_i = k_1 - k_2 \cdot T_i; \quad -\lambda \frac{\partial T}{\partial y} = r_o m_a. \]

The last equation corresponds to the assumption that the heat released as the absorbate flow intersected the interface is consumed only on heating the solution.

Below, as well as in [3], the problem will be solved with the boundary condition, where

\[ m_a = -\frac{\rho D}{1 - C_0} \frac{\partial C}{\partial y}. \]

As it follows from the solution of problems on the film absorption [1,3], for the liquid solution film in conditions of the heat pump, there is the initial segment, where the main temperature change occurs only near the interface inside a thermal layer. Outside this layer within the entrance region the temperature hardly differs from the initial temperature of the solution. In addition, we will consider systems for which the Lewis number \( Le = D/a \) is much less than unity and so the thickness of the diffusion boundary layer is much smaller than that of the thermal layer. Therefore, the asymptotic boundary layer model can be used for calculation of the absorption heat and mass transfer in the entrance region, and conditions outside the boundary layer (both thermal and diffusion layers) can be transferred to infinity and the following conditions hold true \( y \rightarrow \infty; \quad T = T_0, \quad C = C_0 \).

Executing the substitution variable \( x = x + \frac{y^2}{2} \) we can introduce the dimensionless variables

\[ x_1 = \frac{X - a}{u_0 \delta_i^0}; \quad y_1 = \frac{y}{\delta_i^0}; \quad 0 = \frac{T - T_0}{T_e - T_0}; \quad \gamma = \frac{C - C_0}{C_e - C_0}. \]

Here \( C_e = k_1 - k_2 T_0 \) is the equilibrium concentration corresponding to the given initial temperature of the solution \( T_0 \). \( T_e - T_0 \) is the equilibrium temperature corresponding to the given initial concentration of the solution \( C_0 = k_1 - k_2 T_0 \).

By using a self-similar variable

\[ \zeta = y_1 \cdot x_1^{1/4} \]

and dimensionless temperature and concentration variables, the ordinary differential equations can be obtained

\[ \frac{d^2 \theta}{d \zeta^2} + 2 \cdot 3^{2/3} F \cdot r_1^{1/2} \cdot p_{1/2} \cdot \frac{d \theta}{d \zeta} = 0, \]

\[ \frac{d^2 \gamma}{d \zeta^2} + 2 \cdot 3^{2/3} \cdot L \cdot r_1^{1/2} \cdot p_{1/2} \cdot \frac{d \gamma}{d \zeta} = 0 \]

with the boundary conditions

\[ \zeta = 0, \quad \theta + \gamma = 1, \quad \frac{d \theta}{d \zeta} = K_a \frac{d \gamma}{d \zeta}; \]

\[ \zeta \rightarrow \infty, \quad \theta = 0, \quad \gamma = 0. \]

Here, \( Le = \frac{D}{a} \cdot \frac{K_a}{C_e - C_0}; \quad \gamma = \frac{u_0 \delta_i^0}{D} \).

The self-similar solutions of Eqs. (3) and (4) with the boundary conditions (5) and (6) have the form:

\[ \theta = \theta_i (1 - \text{erf}(2 \cdot 3^{2/3} \cdot F \cdot r_1^{1/2} \cdot p_{1/2} \cdot \zeta^{1/4})), \]

\[ \gamma = \gamma_i (1 - \text{erf}(2 \cdot 3^{2/3} \cdot L \cdot r_1^{1/2} \cdot p_{1/2} \cdot \zeta^{1/4})), \]

\[ \theta_i = \frac{K_a L_e^{1/2}}{K_a L_e^{1/2} + 1}, \quad \gamma_i = \frac{1}{K_a L_e^{1/2} + 1}. \]

Note that the formulas (9) for temperature and concentration in the entrance region of the free surface of the film \( \theta_i, \gamma_i \) agree with the formulas obtained for film absorption at other velocity profiles (half parabolic and uniform) [7–9].

In the derivation of the equation for film thickness the condition of conservation of mass in the form \( \frac{D}{a} \int_{0}^{\infty} \rho u \, dy = m_a \) was used and the ordinary differential equation with respect to the film thickness in terms of dimensionless variables was obtained

\[ \frac{d \delta}{d \xi} + \frac{g}{u_0^2 + 2 g x} \delta = -\frac{\lambda}{r_o \delta \sqrt{u_0^2 + 2 g x}} \frac{d T}{d \xi}, \]

defined as the absorbate flow intersected the interface is consumed only on heating the solution.

The solution of this equation with initial condition \( x = 0, \delta = \delta_0 \) in the dimensionless form is

\[ \delta = \frac{\delta_0}{1 + \left( 1 - \frac{2}{3} \cdot L \cdot r_1^{1/2} \cdot p_{1/2} \right) \left( 1 + \frac{K_a \cdot L_e^{1/2}}{C_e - C_0} \right) \left( \frac{x}{\delta_0} \right)^{3/4}}. \]

denoted as the absorbate flow intersected the interface is consumed only on heating the solution.

Thus, the result (10) for the film thickness is valid for both absorption and desorption supposing an appropriate choice of initial values of temperature and concentration. In case of absorption \( \zeta > \zeta_0 \) (film thickness increases due to absorbate influx into the solution), whereas in case of desorption \( \zeta < \zeta_0 \) (film thickness decreases).

For the calculations of temperature and concentration fields within the film it is conveniently to submit the solutions (7) and (8) as

\[ \frac{T}{T_e} = 1 + \theta \left( \frac{T_e - T_0}{T_0} \right); \quad \frac{C}{C_0} = 1 + \gamma \left( \frac{C_e - C_0}{C_0} \right). \]

It is convenient to transform the self-similar variable (2) as

\[ \zeta = \frac{1}{2} \cdot 3^{2/3} \cdot F \cdot r_1^{1/2} \cdot p_{1/2} \cdot \left( 2 \theta \frac{x}{\delta_0} + 1 \right)^{-1/4} \left( \frac{\delta}{\delta_0} - \frac{y^4}{\delta_0} \right). \]

Here \( y^4 \) is measured from the axis of symmetry and varies from 0 to \( \delta \) (\( y^4 = \delta - y \)).

The formulas for calculating the temperatures and concentrations fields are also applicable for both absorption and desorption, depending on the relevant signs \( (T_e - T_0) \) and \( (C_e - C_0) \) in (11).

Fig. 3 shows the temperature and concentration profiles during absorption in several cross-sections of the film in the entrance region. The parameters for the calculations were chosen close to the characteristic values of the two-phase system: water solution of lithium bromide – water vapour, under the assumptions corresponding to the conditions of working absorber of a heat pump. The temperature and concentration profiles were calculated in the film with thickness corresponding to formula (10). Similar results for desorption are shown in Fig. 4.

### 3. Condensation and evaporation heat transfer

The assumptions for heat transfer study during evaporation and condensation are the same as for the absorption study. However, unlike the film absorption, at the film condensation the saturation
temperature at the interface is known and constant for a given pressure of steam.

\[ y = 0, \quad T = T_i, \]
\[ y \to \infty, \quad T = T_0. \]

Eq. (3) is saved for the dimensionless temperature and the boundary conditions have the form:

\[ \xi = 0, \quad \theta = 1, \]
\[ \xi \to \infty, \quad \theta = 0. \]

The solution of Eq. (3) for the dimensionless temperature save the form (7), where \( \theta_i = 1 \).

It is suitable to use formula (12) for self-similar variable in the calculations and figures, and the temperature field inside the film in the form

\[ \frac{T}{T_0} = 1 + \frac{(T_i - T_0)}{T_0}. \]

The dependence of film thickness on the longitudinal coordinate can be obtained:

\[ \frac{\delta}{\delta_0} = \Delta_0 \left[ 1 + \frac{2^{3/4}T_i^{1/4}}{3\pi^{1/2}Pe^{1/2}K^{3/4}} \left( \frac{x}{\delta_0} \right)^{3/4} \right], \]

here \( K = \frac{x}{\sqrt{(T_i - T_0)}}. \)

The obtained solutions for condensation are also valid for the film evaporation.

If \( (T_i - T_0) > 0 \) the film thickness increases due to the condensation, if \( (T_i - T_0) < 0 \), then film evaporates.

Fig. 5 shows the temperature profiles at the different cross-sections of the film within the initial thermal region during the condensation and evaporation for the characteristic parameters of two-phase water-steam system.

Fig. 6 shows the dependence of the film thickness on the longitudinal coordinates during absorption, desorption, condensation and evaporation, and without inflow and outflow of the mass. Differences between the curves in Fig. 6 are not significant for the selected parameters.

Table 1 shows the differences between the calculation results of the obtained dependences. The local Nusselt numbers are shown in Fig. 7 depending on the longitudinal coordinates for different cases.
of mass inflow and outflow in the film (absorption, desorption, condensation, and evaporation). The initial film thickness or that corresponding to the longitudinal coordinate, was chosen as the linear size in the definitions of Nusselt numbers, i.e.,

\[ Nu_1 = \frac{\delta_0}{\nu \rho C_v} \left( \frac{\partial T}{\partial y} \right)_{y=0} \]

\[ Nu_2 = \frac{\delta_0}{\nu \rho C_v} \left( \frac{\partial T}{\partial y} \right)_{y=0} \]

Here \( D_T = \frac{T_i}{\rho C_v} \).

The dependences of the average temperature of the film cross-section on the longitudinal coordinates are shown in Fig. 8(a) for both absorption and desorption, as well as for condensation and evaporation. Similar plots for average concentrations of the film cross-section are shown in Fig. 8(b) for absorption and desorption.

The falling film condensation of pure R12-refrigerant immovable vapour on horizontal smooth cylindrical tube was observed in experimental studies. This film falling regime occurs for large heat fluxes at \( T_i = 70–85 ^\circ C \), when the surface tension of R12 refrigerant decreases. Fig. 9 shows the calculation of the average temperature of the film cross-sections in comparison with the experimental measurements [5,6].

4. Conclusion

The following conclusions can be drawn from the above study. The key criteria characterizing heat and mass transfer in absorption and desorption in the entrance thermal region of the film with free boundaries are complexes Fr, Ka, Pe and Le. Those for condensation are Fr, K and Pe.

The change in film thickness over its length is defined mainly by the Froude number which characterizes the thickness of the film in the absence of mass inflows and outflows.

Absorption, desorption, condensation and evaporation do not change significantly the character of dependence of film thickness on the longitudinal coordinate.

### Table 1
Dimensionless film thickness \( \delta/\delta_0 \) in the absence of the mass inflow/outflow (\( \Delta_0 \)) and in the regimes of film absorption, desorption, condensation, and evaporation.

<table>
<thead>
<tr>
<th>( x/\delta_0 )</th>
<th>( \Delta_0 )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absorption</td>
<td>0.7073</td>
<td>0.7774</td>
<td>0.5</td>
<td>0.4472</td>
<td>0.4082</td>
<td>0.3780</td>
<td>0.3536</td>
<td>0.3333</td>
<td></td>
</tr>
<tr>
<td>Desorption</td>
<td>0.7069</td>
<td>0.5771</td>
<td>0.4997</td>
<td>0.4469</td>
<td>0.4079</td>
<td>0.3776</td>
<td>0.3531</td>
<td>0.3329</td>
<td></td>
</tr>
<tr>
<td>Condensation</td>
<td>0.7102</td>
<td>0.5816</td>
<td>0.5050</td>
<td>0.4528</td>
<td>0.4142</td>
<td>0.3843</td>
<td>0.3602</td>
<td>0.3403</td>
<td></td>
</tr>
<tr>
<td>Evaporation</td>
<td>0.7049</td>
<td>0.5744</td>
<td>0.4965</td>
<td>0.4433</td>
<td>0.4041</td>
<td>0.3735</td>
<td>0.3489</td>
<td>0.3285</td>
<td></td>
</tr>
</tbody>
</table>
It is obvious that the resulting self-similar solutions have limited applicability. Despite these drawbacks, the solutions have certain advantages, particularly in comparison with numerical solutions. The self-similar solutions are simple, because they include the standard functions. Using these solutions allows one to avoid the difficulties associated with the inconsistency of the boundary conditions at the point \((x = 0, y = 0)\). Besides, these solutions applicable for the entrance region can be used as initial conditions for solving the problems beyond the entrance region.

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